Self-Calibrating Measurements of the Density and Velocity of Sound from the Reflection of Ultrasound at Two Solid-Liquid Interfaces

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Design and Objectives

Objectives

- Obtain multiple reflections for both transducers
- Both measurements are self-calibrating
- Determine the reflection coefficients at each solid-liquid interface
- For shear wave multiple reflections, show that experimental values are in agreement with theoretical values
- Use two reflection coefficients to determine the density of the liquid and the velocity of sound in the liquid
Previous Work Using Longitudinal Wave Transducer

- Use multiple reflections for transducer C and determine acoustic impedance \( Z \) of liquid
- Measure time-of-flight (TOF) across pipeline to measure velocity of sound in liquid \( c \)
- Density = \( Z / c \)

References:

Goal: Eliminate TOF measurement by using another transducer at a non-normal incident angle.
Advantage of Using Shear Wave Transducer

Any Angle $\theta$

Angle $\theta < 33^\circ$

Angle $\theta > 33^\circ$
Large RC for $\theta = 45^\circ$

- More Energy in Reflected Wave
- More Echoes
Multiple Reflections for Shear Wave Transducer

911122 50% Sugar Water, SQ wave, 2 gains, 1/9/08

Data: Spectra for 6 SW solutions and water.
5 echoes each → 35 FFTs
Self-Calibration Occurs with Multiple Reflections

- If transducer output changes from one data set to the next, EACH ECHO is affected in the same way.
- Reason: Each echo is dependent upon the very first echo. If the first echo changes, then each subsequent echo changes in the same manner.
- Experimental test, described in *J. Fluids Engineering*, Vol. 126, verifies this description.
- Result: Same transducer output is NOT REQUIRED in two data sets.
Two Data Sets with Shear Wave Transducer Wedge

(a) Data Set 1
Water

(b) Data Set 2
Liquid, Density > 1.0 g/cm³

(c) Water
Frequency (MHz)
FFT Amplitude

(d) Liquid, Density > 1.0 g/cm³
Frequency (MHz)
FFT Amplitude

(e) Water
Frequency (MHz)
Ln (FFT Amplitude)

(f) Liquid
Frequency (MHz)
Ln (FFT Amplitude)
Determining the Reflection Coefficient from Multiple Reflections

- FFT Amplitude $\alpha \ (Reflection \ Coefficient)^{2N}$
- For each echo, there are two reflections at interface.
- Using the same apparatus for water and liquid
  \[ V / W = R^{2N} / R_W^{2N} \]
  where \( R = \) reflection coefficient for liquid
  \( R_W = \) reflection coefficient for water
- Taking the natural logarithm
  \[ \ln (V / W) = 2N \ln (R/R_W) \]
- Linear Relationship: \( Y = N \ \text{slope} \)
  where \( \text{Slope} = 2 \ln (R/R_W) \)
  \( R/R_W = \exp (\text{slope}/2) \) for shear wave reflection
Extension of Method

Previous equation can be written as:

\[ \ln V - \ln W = 2N \ln \left( \frac{R}{R_W} \right) \]

Write this equation for each of the P points shown in graph and sum:

\[ \sum \ln V_i - \sum \ln W_i = 2NP \ln \left( \frac{R}{R_W} \right) \]

\[ [\ln V]_{AV} - [\ln W]_{AV} = 2N \ln \left( \frac{R}{R_W} \right) \]

For each liquid, plot graph of \([\ln V]_{AV} - [\ln W]_{AV}\) versus echo number N and find slope of line.

\[ \frac{R}{R_W} = \exp \left( \text{slope} / 2 \right) \]
Determining Slope for Shear Wave Transducer Data

Slope = -0.103
\[ \frac{R}{R_W} = \exp \left( \frac{\text{slope}}{2} \right) \]
\[ \frac{R}{R_W} = 0.950 \]
P = 5 points
Comparison of Experimental Data with Theoretical Calculations

Reflection Coefficient for 45 Deg Incident shear Wave in Stainless Steel

Next step >>>>>>> Theoretical Calculations
Theoretical Calculations

- When $\alpha_T = 45^\circ$, a reflected longitudinal wave does not exist.
Theoretical Calculations (cont’d)

- When $\alpha_T = 45$, a reflected longitudinal wave does not exist.
  
  $$\sin \alpha_L = \left(\frac{c_L}{c_T}\right) \sin \alpha_T > 1$$

- The angle $\alpha_L$ is imaginary and reflection coefficient $R_{tt}$ is complex.

- Reflection coefficient = real part of $R_{tt}$.

- Details will be available soon in *Proceedings of Meetings in Acoustics*.

- Result: A formulation for $R_{tt}$ is obtained that involves all of the parameters shown in last figure.
  
  $$R_{tt} = f(Z_{liq}, \cos \alpha, \text{known material constants})$$
  
  where $Z_{liq} = \rho c$
Theoretical Calculations (cont’d)

- To compare experimental and theoretical data, use water and sugar water solutions with known density and velocity of sound for the experimental measurements of the reflection coefficient.
- Calculate $R_{tt}$ for all liquids using the formulation using known density and velocity of sound for liquids.
- Plot (or compare) the experimental and theoretical values of
  \[
  \left( \frac{\text{Refl Coeff for Liquid}}{\text{Refl Coeff for Water}} \right)
  \]
  vs. SW concentration
- Excellent agreement, shown on graph, between experimental data and theoretical calculations demonstrates the ability to determine the reflection coefficient experimentally.
Determination of the Density and Velocity of Sound from Two Reflection Coefficients

Use experimental reflection coefficients for the longitudinal wave and that for the shear wave to determine the two unknowns:

- Determine value for experimental reflection coefficient $R_{tt}$ for the shear wave from

  $$R_{tt} = R_{tt} W \cdot \exp \left( \frac{\text{slope}}{2} \right)$$

- Recall that the reflection coefficient for the longitudinal wave led to a determination of the acoustic impedance $Z_{\text{liq}}$ of the liquid.

- Substitute the experimental value of $Z_{\text{liq}}$ into the formulation for the reflection coefficient for the shear wave.

  $$R_{tt} = f(Z_{\text{liq}}), \cos \alpha, \text{known material constants}$$
Determination of the Density and Velocity of Sound from Two Reflection Coefficients (cont’d)

- Solve equation to obtain value of \( \cos \alpha \).
- Knowing the value of \( \alpha \), the velocity of sound \( c \) in the liquid can be obtained from Snell’s Law:
  \[
c = c_T \left( \frac{\sin \alpha}{\sin \alpha_T} \right)
\]
- Density of liquid = \( Z_{\text{liq}} / c \)