Self-Calibrating Measurements of the Density and Velocity of Sound from the Reflection of Ultrasound at Two Solid-Liquid Interfaces

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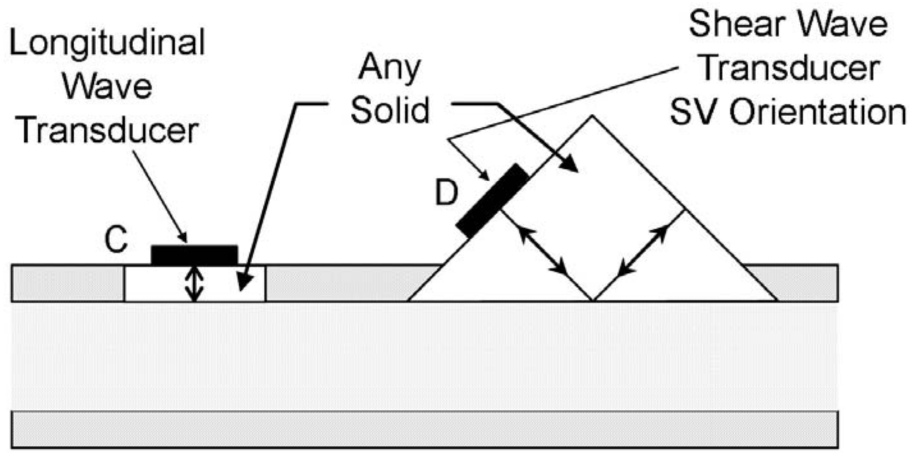
Design and Objectives



- Objectives
  - Obtain multiple reflections for both transducers
  - Both measurements are self-calibrating
  - Determine the reflection coefficients at each solid-liquid interface
  - For shear wave multiple reflections, show that experimental values are in agreement with theoretical values
  - Use two reflection coefficients to determine the density of the liquid and the velocity of sound in the liquid

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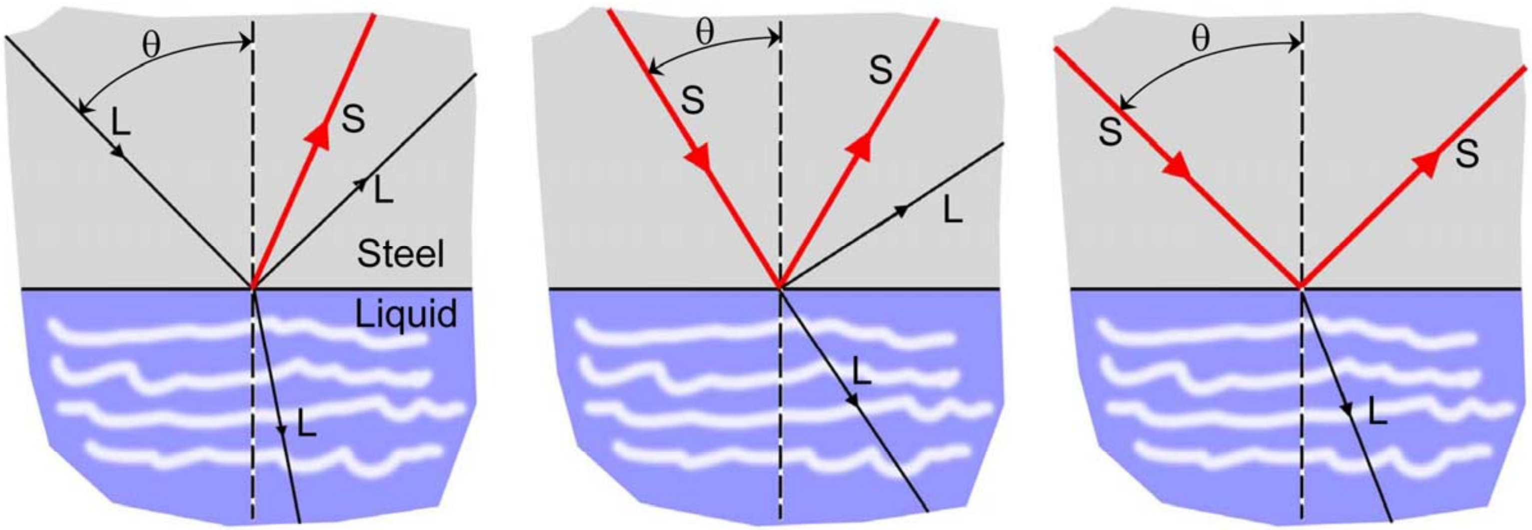
Previous Work Using Longitudinal Wave Transducer

- Use multiple reflections for transducer C and determine acoustic impedance Z of liquid
- Measure time-of-flight (TOF) across pipeline to measure velocity of sound in liquid c
- Density = Z / c
- References:
  - *Journal of Fluids Engineering* **126**, 189-192 (2004)
  - *Ultrasonics* **42**, 563-567 (2004)
- Goal: Eliminate TOF measurement by using another transducer at a non-normal incident angle.

Advantage of Using Shear Wave Transducer



Any Angle θ

Angle θ < 33°

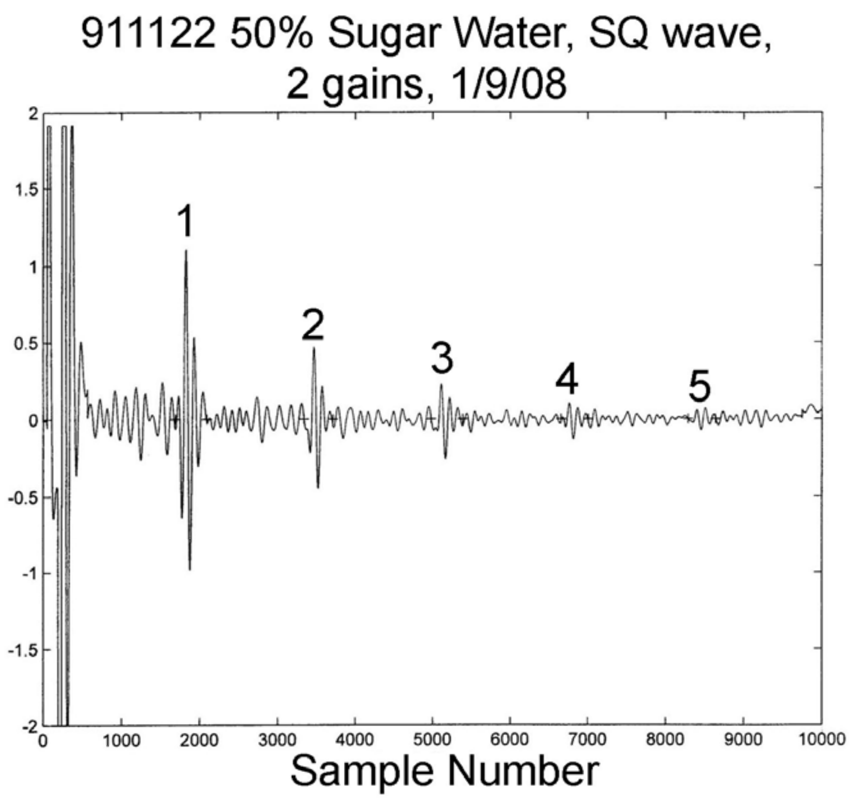
Angle θ > 33°
Large RC for θ = 45°

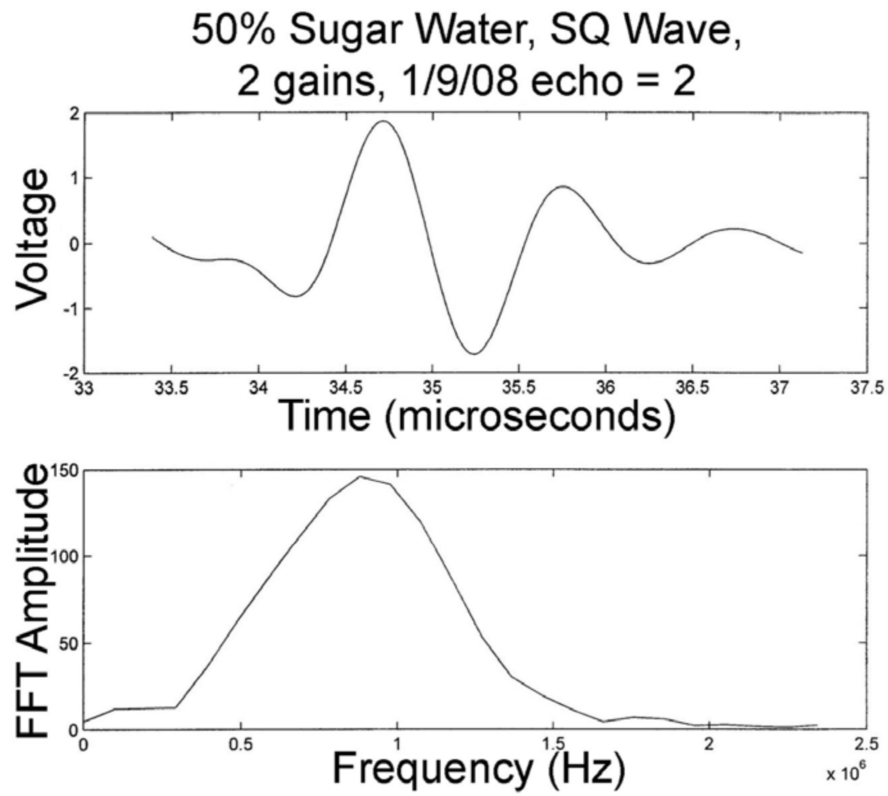
More Energy
in Reflected
Wave

↓

More Echoes

Multiple Reflections for Shear Wave Transducer





Data: Spectra for 6 SW solutions and water.

5 echoes each → 35 FFTs

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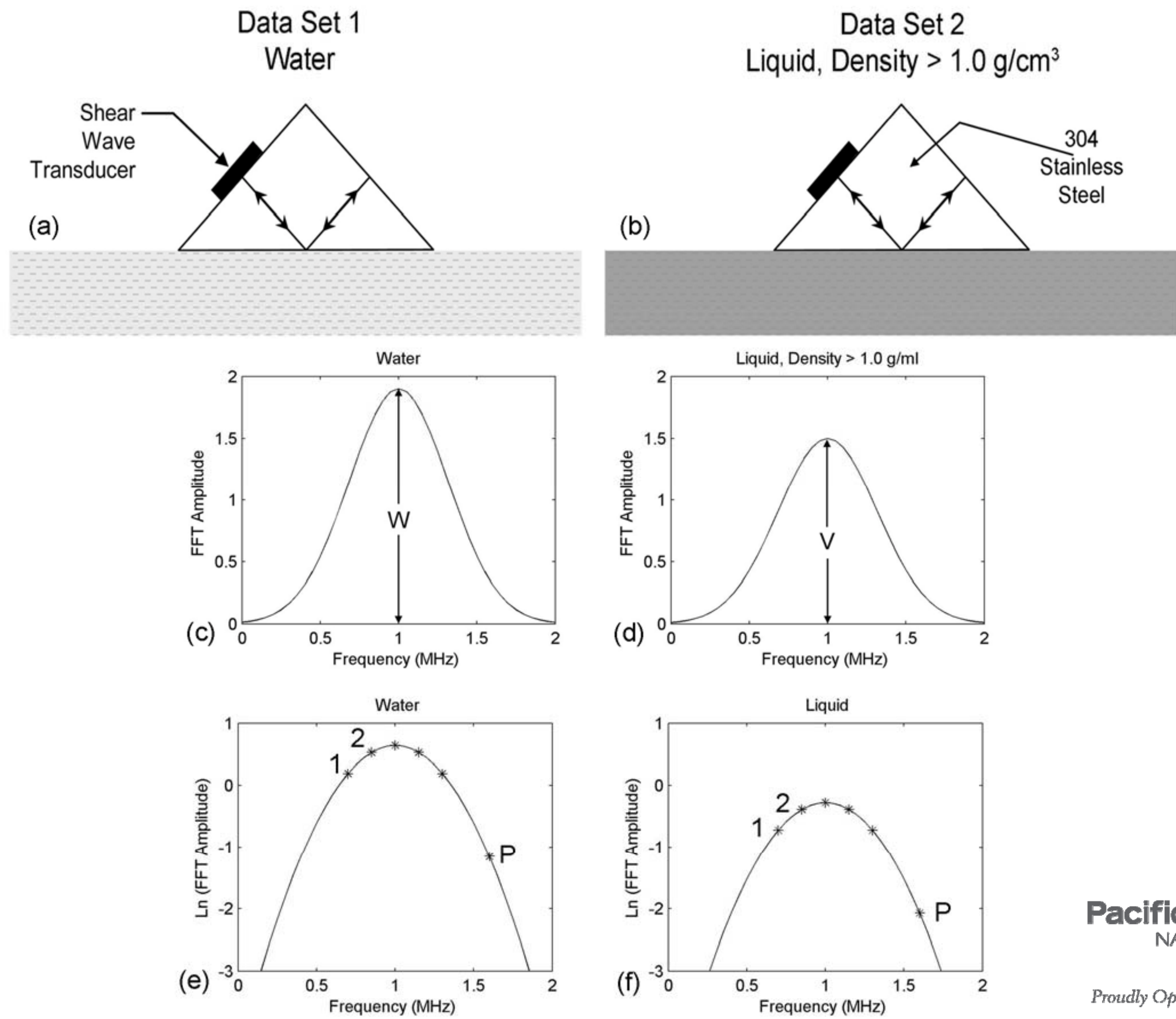
Self-Calibration Occurs with Multiple Reflections

- If transducer output changes from one data set to the next, EACH ECHO is affected in the same way.
- Reason: Each echo is dependent upon the very first echo. If the first echo changes, then each subsequent echo changes in the same manner.
- Experimental test, described in *J. Fluids Engineering*, Vol. 126, verifies this description.
- Result: Same transducer output is NOT REQUIRED in two data sets.

Two Data Sets with Shear Wave Transducer Wedge



Determining the Reflection Coefficient from Multiple Reflections

- FFT Amplitude α (Reflection Coefficient)^{2N}
- For each echo, there are two reflections at interface.
- Using the same apparatus for water and liquid

$$V / W = R^{2N} / R_W^{2N}$$

where R = reflection coefficient for liquid

R_W = reflection coefficient for water

- Taking the natural logarithm

$$Ln (V / W) = 2 N \ln (R/R_W)$$

- Linear Relationship: Y = N   slope

where Slope = 2 ln (R/R_W)

R/R_W = exp (slope/2) for shear wave reflection

Extension of Method

- Previous equation can be written as:

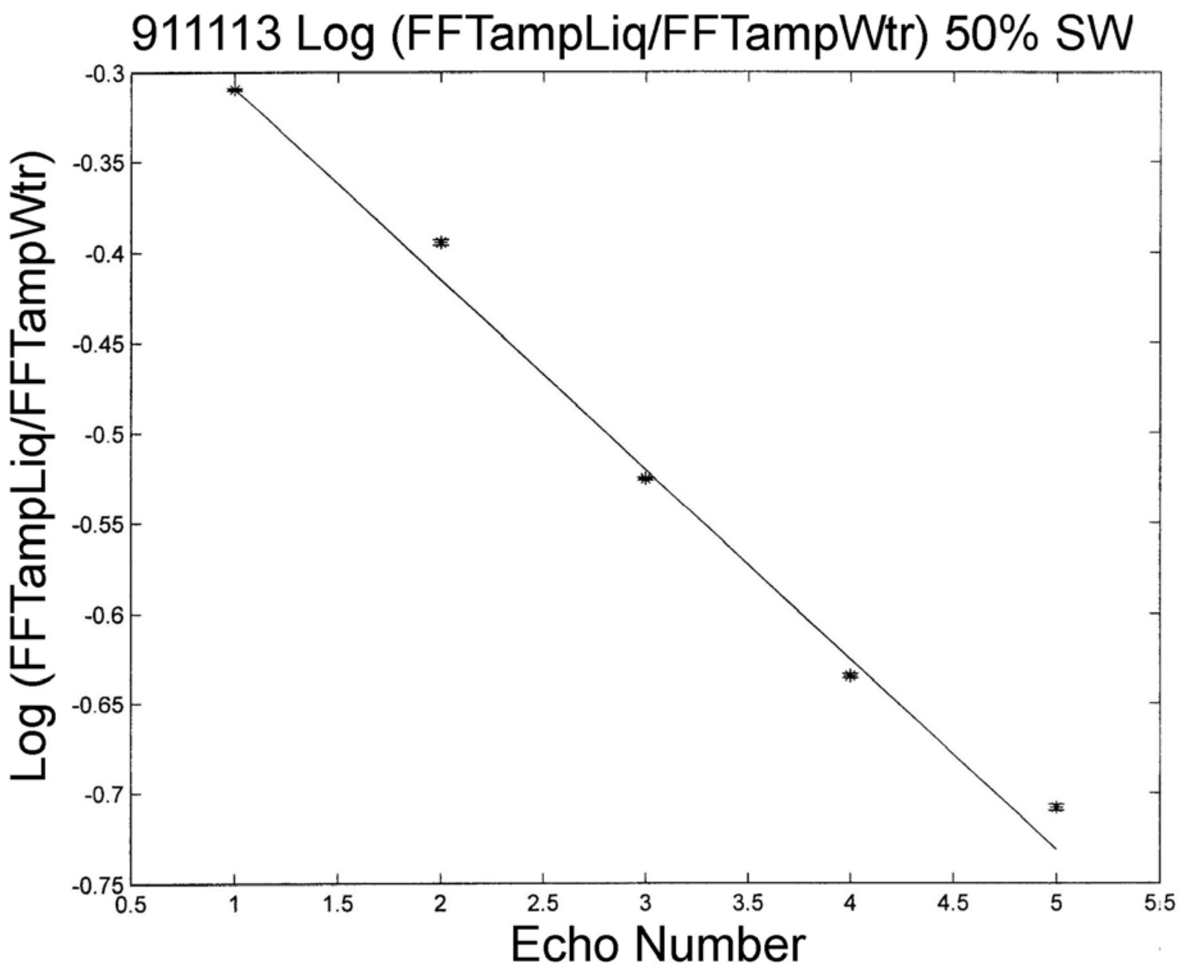
  $$\text{Ln } V - \text{Ln } W = 2\ N\ \text{ln}\ (R/R_W)$$

- Write this equation for each of the P points shown in graph and sum:

  $$\sum \text{Ln } V_i - \sum \text{Ln } W_i = 2\ N\ P\ \text{Ln}\ (R/R_W)$$

- $[\text{Ln } V]_{AV} - [\text{Ln } W]_{AV} = 2\ N\ \text{Ln}\ (R/R_W)$
- For each liquid, plot graph of $[\text{Ln } V]_{AV} - [\text{Ln } W]_{AV}$ versus echo number N and find slope of line.
- $R/R_W = \exp\ (\text{slope} / 2)$

Determining Slope for Shear Wave Transducer Data



Slope = -0.103

R/R_W = exp (slope/2)

R/R_W = 0.950

P = 5 points

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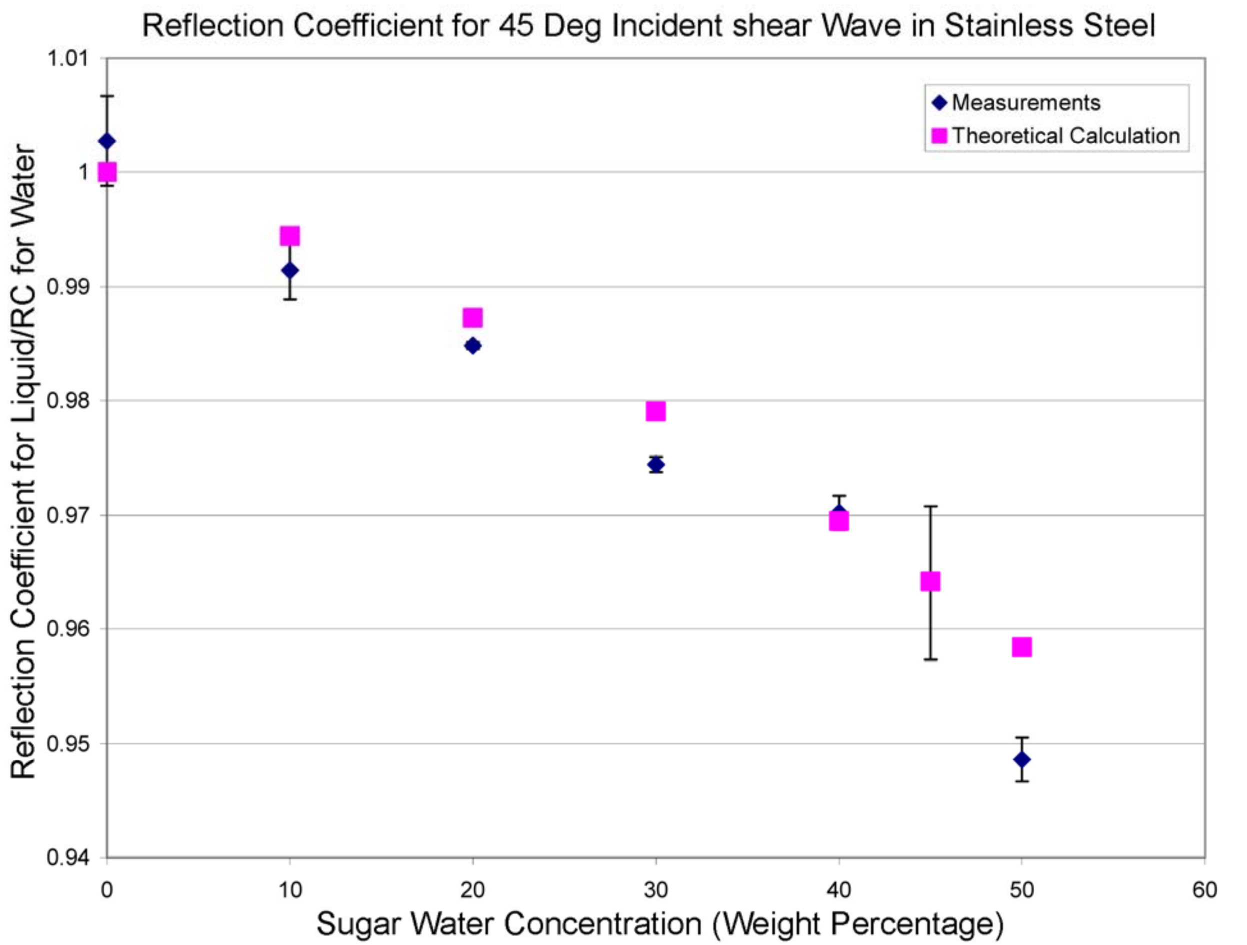
Comparison of Experimental Data with Theoretical Calculations



- Next step >>>>>> Theoretical Calculations

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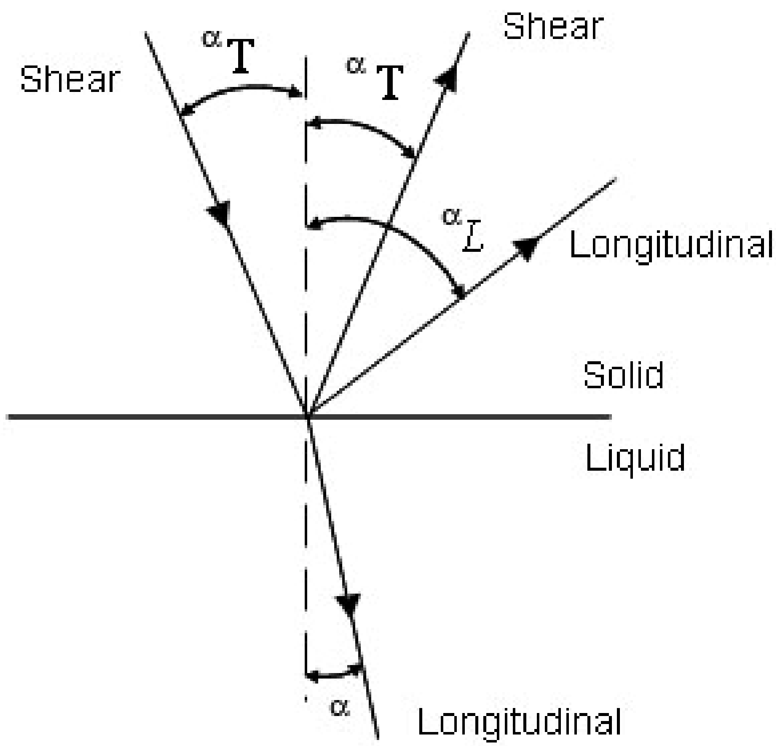
Theoretical Calculations



- Follows discussion in J. Krautkramer and H Krautkramer, *Ultrasonic Testing of Materials*, 4th, p 561-571.
- When α_T = 45 , a reflected longitudinal wave does not exist.

Theoretical Calculations (cont'd)

- When $\alpha_T = 45$, a reflected longitudinal wave does not exist.

  $$\text{Sin } \alpha_L = (c_L/c_T) \sin \alpha_T > 1$$

- The angle α_L is imaginary and reflection coefficient R_{tt} is complex.
- Reflection coefficient = real part of R_{tt}.
- Details will be available soon in *Proceedings of Meetings in Acoustics*.
- Result: A formulation for R_{tt} is obtained that involves all of the parameters shown in last figure.

  $$R_{tt} = f\,(Z_{liq}, \cos \alpha, \text{known material constants})$$

  where $Z_{liq} = \rho\, c$

Theoretical Calculations (cont'd)

- To compare experimental and theoretical data, use water and sugar water solutions with known density and velocity of sound for the experimental measurements of the reflection coefficient.
- Calculate R_{tt} for all liquids using the formulation using known density and velocity of sound for liquids.
- Plot (or compare) the experimental and theoretical values of

$$\left(\frac{\text{Refl Coeff for Liquid}}{\text{Refl Coeff for Water}}\right) \text{ vs. SW concentration}$$

- Excellent agreement, shown on graph, between experimental data and theoretical calculations demonstrates the ability to determine the reflection coefficient experimentally.

Determination of the Density and Velocity of Sound from Two Reflection Coefficients

- Use experimental reflection coefficients for the longitudinal wave and that for the shear wave to determine the two unknowns:
  - Determine value for experimental reflection coefficient Rtt for the shear wave from

    $$R_{tt} = R_{tt}\ W \text{ *exp (slope/2)}$$

  - Recall that the reflection coefficient for the longitudinal wave led to a determination of the acoustic impedance Z_{liq} of the liquid.
  - Substitute the experimental value of Z_{liq} into the formulation for the reflection coefficient for the shear wave.

    $$R_{tt} = f(Z_{liq}), \cos \alpha, \text{ known material constants)}$$

Determination of the Density and Velocity of Sound from Two Reflection Coefficients (cont'd)

- Solve equation to obtain value of cos α.
- Knowing the value of α, the velocity of sound c in the liquid can be obtained from Snell's Law:

$$c = c_T\ (\sin\alpha\ /\sin\alpha_T)$$

- Density of liquid = Z_{liq} / c